Invited Lecture

Specifying Mathematical Language Demands: Theoretical Framework of The Language Specification Grid

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ABSTRACT The didactic perspective on mathematics and language focuses on topic-specific instructional approaches for integrating language learning opportunities into mathematics instruction. From a didactic perspective, a sound and research-based specification of language demands is crucial for providing well-focused learning opportunities. For this, the paper (1) presents the topic-specific specification grid as a useful practical tool for specifying mathematically relevant language demands and (2) explains its underlying theoretical framework by making explicit the four incorporated lenses: epistemic, conceptual, functional, and discursive. The theoretical framework for specifying topic-specific language demands combines various linguistic theory elements and is empirically grounded in findings on typical language demands while mathematics learning.

Keywords. Meaning-related language; Form and function; Conceptual understanding; Discourse practices.

1. Introducing the Didactic Perspective on Language-Responsive Mathematics Teaching and the Need for A Theoretical Framework

Various scientific disciplines have identified students' language proficiency as an important factor for successful mathematics learning: From a psychometric perspective, strong correlations between students' language proficiency and mathematics achievement have been found in assessment studies (e.g., Abedi and Lord, 2001). From a linguistic perspective, these correlations have been explained by exploring the epistemic role of language as a tool for mathematical thinking and knowledge construction (Schleppegrell, 2007). From a sociolinguistic perspective, the language proficiency construct was extended from national languages to social language varieties, with emphasis on the school academic language to which many socially under-privileged students have only limited access (Snow and Uccelli, 2009). From an

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educational classroom research perspective, the language gap in students' mathematics achievement has been explained as an opportunity gap, as students with limited academic language can often only participate peripherally in classroom discourse practices (Herbel-Eisenmann et al., 2011). These findings led to educational calls for enhancing academic language in all subject-matter classrooms (Thürmann et al., 2010).

> The realization of language-responsive instructional approaches for mathematics classrooms still raises many questions, mainly from a didactic perspective: Didactics is the discipline that generates theoretically founded and empirically based knowledge about teaching and learning of a subject matter. It is typical of European didactic traditions that HOW-questions are enriched by deeper topic-specific WHAT-questions (van den Heuvel Panhuizen, 2005). For language-responsive mathematics instruction, these questions are:
>
> - WHAT language demands are most relevant to being treated so that students can benefit most in their mathematics learning?
> - HOW can language learning opportunities be integrated into mathematics classrooms so that students can learn effectively?

The HOW-question has been treated by adopting principles from second-language education (e.g., Gibbons, 2010) and adapting them for mathematics teaching. Subject-specific design research studies, classroom observation studies, and controlled trials have contributed to an empirically grounded set of design principles for language-responsive mathematics instruction (see research overviews in Erath et al., 2021): (1) enhance rich discourse practices, (2) establish a variety of language routines, (3) connect multiple representations and language varieties, (4) include students' multilingual resources, (5) use macro-scaffolding to sequence and combine learning opportunities, and (6) vary and compare language aspects (form, function, etc.) for raising students' language awareness.

However, a targeted realization of these design principles for a particular mathematical topic depends heavily on unpacking what exactly is meant by the overall language proficiency construct. The WHAT-question points to decisions about which language demands can be circumvented and WHAT needs to be explicitly treated as learning content in mathematics classrooms (Bailey, 2007; Moschkovich, 2010; Prediger and Zindel, 2017). Hence, this specification of language demands relevant for a specific mathematical topic is a core didactic challenge for mathematics teachers and for designers of language-responsive instructional approaches and curriculum materials. A recent analysis of US algebra textbooks revealed that in particular for students with low language proficiency, the suggested mathematics and language learning content is poor, often restricted to procedural fluency as a mathematics learning goal and to isolated technical vocabulary as a language learning goal (de Araujo and Smith, 2021). It is the task for didactics as a scientific discipline to develop an empirical and theoretical foundation, based on which designers of language-responsive curriculum materials or teachers can specify language demands that are really relevant for understanding a specific mathematical topic.

This paper presents a practical tool, the specification grid for specifying topic-specific language demands (Section 2), and the underlying theoretical framework that justifies and explains the connections between its elements (Section 3). This theoretical framework combines classical sources in linguistics and language-education research and draws upon subject-specific empirical and design-based mathematics education research focused on language learning.

2. Specification Grid: A Practical Tool for Specifying Language Demands

2.1. Typical pitfalls in specifying language demands for mathematics classrooms

When mathematics teachers' practices start to include language in their mathematics instruction, three typical pitfalls occur in their specifying practices:

- Many mathematics teachers (Turner et al., 2019; Prediger et al., 2019) and curriculum designers (de Araujo and Smith, 2021) start by training isolated vocabulary without connecting it to understanding the mathematics in view.
- Most mathematics teachers exclusively focus on technical language, whereas they falsely assume that important school academic-language demands are already part of students' everyday language (Prediger, 2019; Prediger et al., 2019).
- Some mathematics teachers aim at enhancing discourse practices (e.g., discuss-ing multiple solutions for a calculation task), but not those that are most critical for developing students' conceptual understanding (especially explaining meanings and describing general patterns; Setati, 2005; Erath et al., 2018; Prediger et al., 2019)

In all of these cases, teachers can invest a lot of energy and classroom time to teach students to master a specified language demand, but when the language demands themselves are peripheral to mathematical understanding, they distract rather than support students' mathematics learning. That is why Moschkovich (2015) pleaded for a focus on discourse practices rather than vocabulary, and Setati (2005) emphasized the need for conceptual rather than procedural talk.

2.2. The specification grid for language demands as a practical tool

To overcome these often-documented pitfalls, we developed a practical tool named "the specification grid" that can support mathematics teachers, curriculum designers, and researchers to specify mathematically relevant language demands (Fig. 1, see in next page).

The specification grid incorporates four lenses that foreground different components of the grid and their interplay that are to be explained, connected to the grid, and then theoretically founded (Section 3):

- Epistemic lens: Language as a thinking and learning tool;
- Conceptual lens: Focus on conceptual understanding of mathematics;
- Functional lens: Double use of form-function relationship, where language is viewed as serving particular purposes and not viewed only as forms;
- Discursive lens: Discourse practices as the essential language learning content.

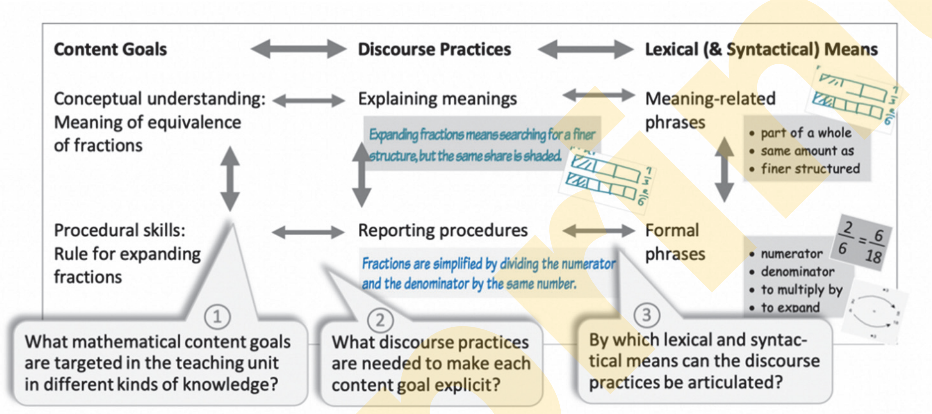

Fig. 1. Specification grid for identifying mathematical language demands (Prediger, 2019)

The epistemic lens entails that the specification question is posed as "What language demands are most relevant to being treated so that students can benefit most in their mathematics learning?" This means that we do not only focus on the communica-tive function of language as a medium of communication, but also on the epistemic function of language as a thinking and learning tool (Pimm, 1987; Snow and Uccelli, 2009) for mathematics learning, not a generic academic-language proficiency. In the specification grid, the epistemic lens is incorporated by determining the relevance of particular language demands from their role for learning particular mathematical content goals. Practically, this is realized by starting the specification process in the first column of Fig. 1, by setting the mathematical content goals of a particular teaching unit; in this example it is the equivalence of fractions.

The conceptual lens is incorporated in the specification grid by the rows that distinguish between conceptual understanding and procedural skills. Both kinds of knowledge are relevant in mathematics education, but procedural skills still tend to be prioritized in classroom practices (Hiebert and Grouws, 2007). In our practical specification example in Fig. 1, Question 1 leads to distinguishing understanding the meaning of the equivalence of fractions from the procedure of expanding fractions.

The discursive lens treats the discourse practices as the key language unit in view. The functional lens is incorporated into the specification grid by two form-function relationships: For the epistemic function of expressing the procedural skills and conceptual understanding, the discourse practices provide the key language units. In the practical example, Question 2 leads to specifying the discourse practice of “reporting procedures” for articulating the procedural knowledge and the very distinct discourse practice of “explaining meanings” for articulating the conceptual understanding. Question 3 focuses the second form-function relationship of lexical and syntactical means needed for realizing the discourse practices in phrases.

In an empirical study on teachers’ specification practices for the case of equivalence of fractions (Prediger, 2019), most teachers immediately identified the typical formal vocabulary such as numerator, denominator, expand, and multiply as relevant for expressing the equivalence of fractions, but without being aware of their functional connections, in other words, that these phrases can only be used in the discourse practice of reporting procedures that can underpin the procedural skill. Much less often, the teachers specified the discourse practice of explaining meanings, although it is epistemically relevant for developing conceptual understanding. Teachers were not aware that explaining meanings requires other phrases to express mathematical structures and relationships such as “describes the same share, but more coarsely structured,” that we have termed meaning-related phrases (Pöhler and Prediger, 2015). Meaning-related phrases can require new academic vocabulary, but also syntactical means for expressing complex relationships, for example, binary relations for comparisons (“this number is larger than this number” rather than “this number is large and this number is small”) or sophisticated syntactical constructions such as “increases more slowly” with adverbs fine tuning the meaning of verbs (Prediger and ŞahinGür, 2020).

Summing up, the specification grid supports specifications of mathematical and language learning content in an overall epistemic lens. The vertical arrows in the specification grid in Fig. 1 make explicit the relevant distinction related to procedural and conceptual aspects in a conceptual lens, and the horizontal arrows incorporate the functional lens entailing important functional connections between content sub-goals, the discourse practices for each sub-goal, and the lexical or syntactical means to realize the discourse practices. Positioning the discourse practices in the middle column reflects their relevance in a discursive lens.

In a PD research project, we showed that with the support of the specification grid, mathematics teachers learned to identify mathematically relevant language demands with a higher accuracy (ŞahinGür and Prediger, 2019), so it turned out to be practically useful. In the next section, the theoretical backgrounds of the lenses are described, together with the empirical findings strengthening the claims underlying the postulated connections and distinctions of vertical and horizontal arrows.

3. Four Lenses Underlying the Specification Grid and Their Background

Without aiming at a comprehensive account of all perspectives on language and mathematics learning (as provided by Morgan et al., 2014, or the early book by Pimm, 1987), this section explains the theoretical framework for the practical purposes sketched in Section 2, led by four lenses, each intertwined in pairs.

3.1. Backgrounds for the functional and epistemic lenses: Studying language in its epistemic function for students' knowledge construction processes

Whereas linguistic research sometimes studies language as a form with relevance in itself (e.g., different lexical or syntactical phenomena), mathematics education research on language has adopted a functional lens from the beginning, that is, the function of language for and in mathematics teaching and learning was considered. Early on, Austin and Howson (1979) and Pimm (1987) described the two major functions of language for mathematics classrooms: the communicative function as a tool for classroom interaction and the epistemic function of language as a tool for thinking and learning. Epistemic, in this context, means related to students' individual or collective knowledge construction processes. Vygotsky (1934) explains the epistemic role of language by the relevance of interiorizing external operations (learned in social interaction) by inner language. The epistemic lens can be considered as a first-level realization of the functional lens on language. The sketched Vygotskyan theoretical background, however, does not sufficiently help to disentangle what aspect of language is relevant. Accordingly, Morgan et al. (2014) promoted the specification question "What are the linguistic competencies…required for participation in mathematical practices?" (p. 851) as crucial for future research. Its systematic treatment requires further lenses.

3.2. Epistemic and conceptual lenses: Language as a thinking and learning tool for developing conceptual understanding

As Vygotsky (1934) and Cummins (1979) had already pointed out, the epistemic function of academic language is particularly relevant for higher order thinking skills and for understanding abstract scientific concepts, whereas more elementary ideas and concepts can be learned with less elaborate language. More recent linguistic and language-education theories confirm this connection of elaborateness of language and thought, and even define academic language by its epistemic function: Academic language is "the language that is used by teachers and students for the purpose of acquiring new knowledge and skills…, imparting new information, describing abstract ideas, and developing students' conceptual understanding" (Chamot and O'Malley, 1994, p. 40).

These theoretical backgrounds underpin our MuM research group's decision to focus the epistemic lens on language mainly on a conceptual lens, in other words, for the development of conceptual understanding. This decision was fueled by empirical findings that language proficiency has more of an impact on the conceptual understanding than procedural skills, as shown by studies in Grade 3 (Ufer et al., 2013) and Grade 10 (Prediger et al., 2018). Even if concep-tual understanding and procedural skills must always be developed in mutual dependence (Kilpatrick et al., 2001), specifying language demands with a conceptual lens thereby focuses on those types of knowledge that pose more language challenges for teachers and students, namely, conceptual understanding.

In the theoretical framework, the adopted conceptual lens draws upon defining conceptual understanding as grasping the meaning of concepts. Meanings are not Platonist ideas, but socially constructed networks of mental representations. Hiebert and Carpenter's (1992) definition of the meaning of a mathematical concept was that it is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and the strength of the connections. […] [It] is understood thoroughly if it is linked to existing networks with stronger or more numerous connections. (Hiebert and Carpenter, 1992, p. 67)

This characterization of conceptual understanding as a network with strong connections has been further elaborated in at least three ways: (a) with respect to multiple external representations, (b) with respect to concept elements, and (c) with respect to the epistemic and semiotic processes required to build the networks.

(a) In many instructional approaches, multiple external representations count as crucial tools for developing conceptual understanding. Indeed, the translation from manipulatives to graphical representations to symbolic representations has proven effective for developing conceptual understanding (Lesh, 1979), in particular when the representations are not only juxtaposed, but explicitly connected (Renkl et al., 2013). The focus on connecting rather than only implicitly translating is in line with Hiebert and Carpenter's (1992) emphasis on connections as characteristic in understanding. With respect to language in mathematics learning, the emphasis on manipulatives and graphical representations must not be mistaken as a substitute for explicit verbalizations. Instead, various empirical studies have shown that explicit negotiations of mean-ings are required before students "see" the relevant structures in graphical representations (Meira, 1998; Steinbring, 2005).

(b) Beyond external representations, Hiebert and Carpenter (1992) pointed to knowledge elements that are to be connected in the mental representation of a mathematical concept or theorem. These knowledge elements in the connected networks can comprise concept elements (e.g., a particular basic mental model or a subconstruct of a concept) and procedural elements (e.g., the procedure of expanding fractions by multiplying numerator and denominator by equal numbers is connected to graphically structuring a fraction bar into a finer structure; Korntreff and Prediger, 2022).

(c) Based on this characterization of conceptual understanding as a network of mental representations of knowledge elements in multiple external representations, we distinguish four epistemic processes that are needed to develop understanding (Prediger and Zindel, 2017; Korntreff and Prediger, 2022): Students

- mentally construct knowledge elements (e.g., by explicating them in the semiotic processes of translating or connecting multiple external representations),
- connect several knowledge elements into a network,
- compact the knowledge elements or external representations into new conceptual entities which can then serve as new elements of higher order, and
- unfold the compacted concepts into their constituent knowledge elements when necessary (e.g., for justification or explaining connections).

To unpack the inverse epistemic processes compacting and unfolding, our theoretical framework draws upon Drollinger-Vetter's (2011) interpretation of Aebli's (1981) conceptualization, who describes compacting as the highly relevant epistemic process in which networks of (internal or external representations) and knowledge elements are encapsulated into new conceptual entities, which can then be the elements for networks of higher complexity. Successful compacting processes can be reversed, that is, the encapsulated concept or procedure can be unfolded back into the constituent knowledge elements. In our research, we showed that for many students, this reversibility is only fragile or not achieved, and requires elaborated discourse practices with concise phrases (Prediger and Şahin-Gür, 2020).

3.3. Discursive and functional lens: discourse practices and means to enact them

In sociolinguistics, the differences between everyday language and academic language have been characterized by different dimensions: in the lexical dimension (e.g., by specialized vocabulary, composite or unfamiliar words, and specific connectors), in the syntactical dimension (e.g., long and syntactically complex sentences, passive voice constructions, and long noun phrases and prepositional phrases), and in the discursive dimension (e.g., turn-taking organization, situative language use, and subject-specific text types; Snow and Uccelli, 2009; Heller and Morek, 2015).

Most mathematics education researchers have emphasized that the atomistic lexical and syntactical dimensions must be subordinated to the discursive dimension, as the mathematical discourse is the major language unit relevant for mathematics learning (Adler, 2001; Moschkovich, 2010; Setati, 2005; Herbel-Eisenmann et al., 2011). This widely agreed discursive focus has been adopted with many different theoretical backgrounds: Ryve's (2011) research overview identified a huge variety of conceptualizations of discourse, spanning from complex culturalistic or conversational perspectives to commognition perspectives. Here, we follow Moschkovich (2010, 2015) in her emphasis on the discursive lens in the epistemic function for meaning-making and her conceptualization of mathematical discourse that "draws on hybrid resources and involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages and the everyday register)" (Moschkovich, 2015, p. 2). To further narrow the discursive lens down, we base our discursive lens on interactional discourse analysis (Quasthoff et al., 2017), in which patterns of discourse are considered as socio-culturally evolved and then interactively co-constructed in classroom discourses (on the micro-sociological level).

The key units of language in interactional discourse analysis are discourse practices, which are

> defined as multi-unit turns … interactively co-constructed, contextualized and functionally oriented towards particular genres such as narration, explanation or argumentation. By making use of conventionalized genres, discourse units in their joint achievement in interaction rely on patterns available in speech communities' knowledge. (Erath et al., 2018, p. 4)

Typical discourse practices are narrating, explaining, arguing, and reporting. As Morek and Heller (2015) explained, academic discourse practices are those optimized by school purposes, in particular explaining what and explaining why to convey or construct knowledge and arguing to negotiate divergent validity claims in classrooms.

The theoretical construct of discourse practices resonates with the constructs "language actions within text genres" (explaining, describing, and evaluating; Roll et al., 2019) and "discourse functions" (classifying, defining, describing, evaluating, reporting, explaining, and exploring; Thürmann et al., 2010; Dalton-Puffer et al., 2018). The three constructs overlap in (a) the adopted epistemic lens that conceives cognitive processes and language practices as tightly connected; (b) the characterization as pattern of language in discursive dimension, serving for certain typical communicative and epistemic purposes; and (c) the educational emphasis on their dual nature as being learning medium and learning goal. The two constructs differ substantially in that discourse practices and discourse functions are explicitly also related to oral language (whereas the language actions in text genres are restricted to written language) and in that discourse practices are characterized, not in psycholinguistic ways as individual mental practices, but in sociolinguistic ways by their interactive co-constructed nature.

Whereas Roll et al. (2019), Bailey (2007), and Thürmann et al. (2010) identified their lists of discourse functions/language actions by analyzing textbooks and written curricula, Dalton-Puffer et al. (2018) and our studies (Prediger and Zindel, 2017; Erath et al., 2018; and others) investigated transcribed mathematical teaching learning processes to identify the relevant oral and written discourse practices.

Tab. 1. Empirically identified list of the most important mathematical discourse practices

Mathematical discourse practice	**Explanation**
Naming	• Stating numbers/results, naming words • Assigning individual words/information/elements to something without an explanation
Narrating	• Non-condensed narratives of everyday experiences, mostly organized sequentially without extracting any mathematical structure
Reporting procedures	• Reporting individual procedures in sequential but concrete ways (e.g., previous solution paths) • Elucidating general procedures in sequential but generalized ways
Explaining meanings	• Interpreting a concept/formal element in graphical representations, contexts, etc. • Articulating how two external representations are connected
Arguing	Justifying a connection by reducing to aspects established as true, e.g., • justifying choices of representations by referencing structural elements • justifying choices of operations by referencing their meanings • refuting a conjecture by providing counterarguments
Describing mathematical structures	• Articulating the structure of a context situation, e.g., a functional relationship or part-whole relationship
Describing general relationships	• Example-oriented (generic) verbalization of relationships • General verbalization of relationships (e.g., with word variables)
Evaluating	• Formulating/justifying an independent evaluation judgment about facts by drawing upon mathematical knowledge/reasoning • Forming an opinion

For the didactic perspectives adopted in the MuM research group (with its focus on what-questions; see Section 1), the discourse practices as characterized by interactional discourse analysis (Erath et al., 2018; Quasthoff et al., 2017) needed to be refined by what is treated. Tab. 1 lists the repeatedly iden-tified relevant discourse practices.

Some researchers of discursive dimensions have pleaded for focusing the discursive dimension instead of the lexical and syntactical dimension (e.g., Barwell, 2012). But this separation of forms and functions has already been problematized by Solano-Flores (2010). He sketches four research traditions: two focusing on language in its function for mathematics learning (language in its epistemic function as a process in investigating development and cognition, e.g., for meaning-making, and language more in its communicative function as a system in investigating social interaction) and two traditions focusing on language forms (language as a structure, studied regarding lexical or syntactical difficulties in tests, and language proficiency as a factor investigated with respect to the achievement of different student groups). Solano-Flores (2010) pleaded for combining the four traditions to capture more complex language issues.

This combination is realized in functional linguistic perspectives when considering discursive, syntactical, and lexical dimensions in their functional connections:

> Lexical and morpho-syntactical forms prevalent in academic texts… are … made for…presenting information in highly structured ways… that enable the author/speaker to take an assertive, expert stance toward the information presented.…The high frequency of nominalizations and expanded noun phrases… can be explained by their functions…for knowledge transfer:…avoid ambiguity…condensing previously given information. (Heller and Morek, 2015, p. 176; see also Schleppegrell, 2007)

In our functional lens, we follow Moschkovich (2015) and interactional discourse analysis (Heller and Morek, 2015) to subordinate the dimensions, not by priority but functionally. This means that lexical and syntactical dimensions are considered for identifying the necessary means to textualize and mark the discourse practices. For example, topic-independent lexical means for reporting procedures comprise temporal connectors for marking the sequential structure (e.g., "at first," "then," "later," "finally"), whereas explaining meanings or arguing requires integrating connectors (e.g., "for this," "because of," "this means"). Realizing discourse practices with sequential structure is easier for most students than realizing discourse practices with integrative structure and global coherence, because the integration of structures also requires a mental condensation of ideas and more condensed phrases (Schleppegrell, 2007; Erath et al., 2018). These necessities explain why reporting procedures is enacted more successfully by many students than explaining meanings (Erath et al., 2018).

Summing up, specifying the relevant discourse practices needed to articulate a particular mathematical learning goal allows researchers and designers to specify relevant language demands. Discourse practices are well-defined units in the discursive dimension and carry with them lexical and syntactical features as means for engaging in them.

3.4. Discursive and conceptual lens: Discourse practices of explaining meanings and meaning-related phrases as the essential language learning content

Combining a discursive and conceptual lens, Setati (2005) already hinted at the problem that most discourses in her observed classrooms were shaped mainly by procedural talk and rarely by conceptual talk. Many other researchers have similarly problematized that in classrooms with mainly procedural talk, students find too few learning opportunities for conceptual understanding. Hence, the discourse practices involved in collective meaning-making were analyzed in depth (Moschkovich, 2010; Barwell, 2018), yet so far mainly without using these analyses for identifying the topic-specific language demands involved in realizing conceptually strong discourses.

From our particular perspective on discourse practices in a functional and discursive lens, the distinction between procedural and conceptual talk is reflected by the distinction between two rows in the specification grid (Fig. 1). The functional connection makes visible that the discourse practices of reporting procedures can mainly support the learning of procedural skills, whereas the development of conceptual understanding of mathematical concepts requires the discourse practice of explaining meanings (Pöhler and Prediger, 2015; Prediger and Zindel, 2017). For practical purposes, the distinction into these two discourse practices is sufficient and insightful for professional development of teachers (e.g., Prediger, 2019).

For research purpose and subtler design decisions, however, the distinction can be further refined by more in-depth analyses of the semiotic and epistemic processes introduced in Section 3.2 and their verbalization in discourse practices, in particular for collectively unfolding compacted concepts and external representations. Three epistemological and ontological characteristics of mathematical concepts shape the necessity of elaborate discourse practices and elaborate lexical means for the epistemic processes of developing their understanding:

a) Mathematical concepts are abstract. As the meaning of mathematical concepts cannot simply be grasped by pointing to external objects (“This is a table.”), language is required for negotiating meanings of abstract entities. Therefore, the relevance of multiple representations in the interactive processes of mean-ing constructions have been outlined (e.g., Lesh, 1979).
b) Mathematical concepts are relational in nature: Steinbring describes “The particular epistemological difficulty of mathematical knowledge — contained in the specific role of … signs and symbols — consists in the fact that mathematical knowledge does not simply relate to given objects, but also that relations, structures and patterns are expressed in it” (Steinbring, 2005, p. 4; similar Barwell, 2018).
c) Mathematical knowledge elements must be connected in dynamic networks. Whereas students quickly acquire temporal connectors (e.g., “first,” “after that,” “then”), the connections that have to be made explicit when building and connecting knowledge elements into networks of understanding require more elaborate connectors.

Whereas many authors have expressed the hope that processes of meaning-making can and should completely rely on the individual resources that students bring into the mathematics classroom, these particular epistemological and ontological characteristics of mathematical concepts pose particular challenges when articulating ideas in the four epistemic processes of constructing, connecting, compacting and unfolding mathematical concepts. In particular, the process of unfolding requires not only vague, ambiguous, and perhaps deictic everyday resources (e.g., “this, here,” “there, you know”), but also academic phrases for realizing concise and explicit explanations of meanings.

With meaning-related phrases, our research group established a new construct that encompasses all topic-specific lexical (and sometimes also syntactical) means required to express the abstract and relational nature of a particular concept in an explicit and concise yet informal way. Although some students with high academic-language proficiency might have sufficient individual meaning-related phrases in their individual resources, students with lower academic-language proficiency have been shown to lack exactly these meaning-related phrases. For example, even many seventh graders fail to crack complex percent information and connect the part and the whole only with "and," without explicitly expressing the part-whole relationship, using, for example, "out of" (Pöhler and Prediger, 2015), and many 10th graders struggle with coordinating two quantities in functional relationships as they cannot articulate "the price depends on the weight" or "the price grows with the weight" (Prediger and Zindel, 2017). The topic-specific meaning-related phrases have thereby turned out to be a key area of language-learning content to enable all students to engage in demanding discourse practices such as explaining meanings and arguing. In classroom interaction, establishing shared meaning-related phrases also strengthens the possibilities for joint knowledge construction processes (Prediger and Pöhler, 2015).

3.5. Outlook: epistemic and discursive lenses in depth and for designs

Although our investigations of processes of developing conceptual understanding have already substantially contributed to specifying the mathematically relevant language demands, still further research is necessary to deepen the empirical exploration of the epistemic and discursive lenses. In our current research, we investigate how the four epistemic processes (mentally construct knowledge elements, connect knowledge elements, compact knowledge elements, and unfold compacted knowledge elements) are articulated in the discourses and what students' major language challenges are when engaging in these epistemic processes.

We can provide quantitative evidence that classrooms in which teachers engage students in rich discourse practices and constantly connect different knowledge elements have significantly higher learning gains than others without these rich discourse practices and important epistemic processes (Neugebauer and Prediger, submitted). Further qualitative research can reveal deeper insights into the underlying mechanisms.

This, in turn, can also inform the design of language-responsive mathematical learning opportunities, which should be the overall goal of research.

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